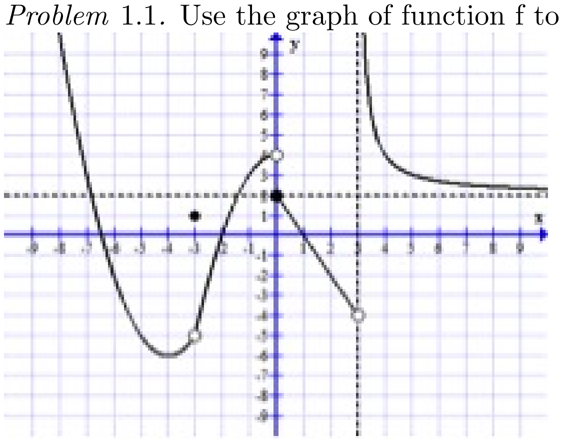
CALCULUS I: FIU FINAL EXAM PROBLEM COLLECTION: VERSION 04.35 WITHOUT ANSWERS

FIU MATHEMATICS FACULTY NOVEMBER 2017

Contents

1. Limits and Continuity

Problem 1.1. Use the graph of function f to find the limits



(1) $\lim_{x\to -3^-} f(x) =$

(2) $\lim_{x\to -3^+} f(x) =$

(3) $\lim_{x\to -3} f(x) =$

(4) $\lim_{x\to 0^-} f(x) =$

(5) $\lim_{x\to 0^+} f(x) =$

(6) $\lim_{x\to 0} f(x) =$

(7) $\lim_{x\to 3^-} f(x) =$

(8) $\lim_{x\to 3^+} f(x) =$

(9) $\lim_{x\to 3} f(x) =$

(10) $\lim_{x\to -4^-} f(x) =$

(11) $\lim_{x\to -4^+} f(x) =$

(12) $\lim_{x\to -4} f(x) =$

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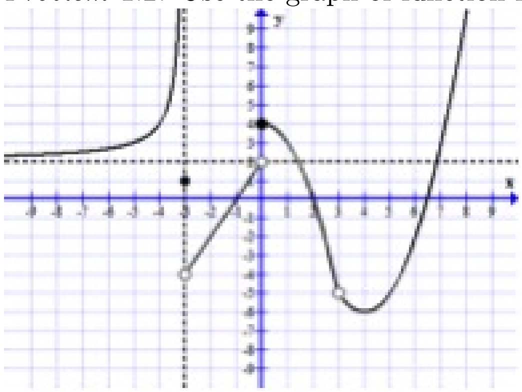
Problem 1.2. Use the graph of function f to find the limits



(1) $\lim_{x\to -3^-} f(x) =$
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(7) $\lim_{x\to 3^-} f(x) =$
(8) $\lim_{x\to 3^+} f(x) =$
(9) $\lim_{x\to 3} f(x) =$
(10) $\lim_{x\to 6^-} f(x) =$
(11) $\lim_{x\to 6^+} f(x) =$
(12) $\lim_{x\to 6} f(x) =$

Problem 1.3. Find the limits. If the limit does not exist, then write dne

(1) $\lim_{x\to 2} \frac{2x^2-3x+2}{x^2+4x+4}$
(2) $\lim_{x\to 2} \frac{2x^2-3x-2}{x^2+4x+4}$
(3) $\lim_{x\to 2} \frac{2x^2-3x+2}{x^2-4x+4}$
(4) $\lim_{x\to -\infty} \frac{1-x^3}{1+x^2}$
(5) $\lim_{x\to 3} \frac{x^4-81}{x^2-7x+12}$
(6) $\lim_{x\to 3} \frac{9-x^2}{3-x}$
(7) $\lim_{x\to 6^+} \frac{x-3}{x^2-x-30}$
(8) $\lim_{x\to 2} \frac{x}{|2-x|}$
(9) $\lim_{x\to -\infty} \frac{\sqrt{2x^2-3}}{x-2}$
(10) $\lim_{x\to \infty} \left(\sqrt{x^2+2x}-x\right)$
(11) $\lim_{x\to \infty} \left(\sqrt{x^2+5}-x\right)$
(12) $\lim_{x\to 1} \cos^{-1}(\ln x)$
(13) $\lim_{x\to 0} \frac{1-\cos(4x)}{x}$
(14) $\lim_{x\to \infty} \frac{e^{2x}}{x^2}$
(15) $\lim_{x\to \infty} \frac{\ln(2x)}{e^{3x}}$
(16) $\lim_{x\to 0} \frac{\tan(4x)}{\sin(3x)}$
(17) $\lim_{x\to 0} \frac{\sin(x)}{x(x-4)^2}$
(18) $\lim_{x\to 0} \frac{\arcsin(3x)}{\sin(x)}$
(19) $\lim_{x\to 0} \frac{x^2}{1-\cos^2 x}$
(20) $\lim_{x\to \infty} xe^{-x}$
(21) $\lim_{x\to -\infty} xe^{-x}$
(22) $\lim_{x\to \infty} x\sin\left(\frac{\pi}{x}\right)$
(23) $\lim_{x\to \left(\frac{\pi}{2}\right)^-} (\tan(x)-\sec(x))$
(24) $\lim_{x\to \frac{\pi}{4}} (1-\tan x)\sec(2x)$
(25) $\lim_{x\to 0^+} \left(\frac{1}{x}-\csc(x)\right)$
(26) $\lim_{x\to 0^+} x^x$
(27) $\lim_{x\to 0^+} (1+4x)^{1/x}$

Problem 1.4. Let $f(x) = \begin{cases} 0, & if\ x \leq -5 \\ \sqrt{25-x^2}\,, & if\ -5 < x < 5 \\ 3x\,, & if\ x \geq 5 \end{cases}$. Find the indicated limits

(1) $\lim\limits_{x\to -5^-} f(x) =$
(2) $\lim\limits_{x\to -5^+} f(x) =$
(3) $\lim\limits_{x\to -5} f(x) =$
(4) $\lim\limits_{x\to 5^-} f(x) =$
(5) $\lim\limits_{x\to 5^+} f(x) =$
(6) $\lim\limits_{x\to 5} f(x) =$
(7) Is f continuous at 5?
(8) Is f continuous at -5?

Problem 1.5. Let $f(x) = \begin{cases} \frac{4x}{5x+1}, & if\ x < 1\,000\,000 \\ \frac{5x}{4x+1}\,, & if\ x \geq 1\,000\,000 \end{cases}$. Find the indicated limits

(1) $\lim\limits_{x\to -\infty} f(x) =$
(2) $\lim\limits_{x\to +\infty} f(x) =$
(3) Does the function have any asymptotes? If yes, how many? Provide the equations of the asymptotes if there is any.

Problem 1.6. Locate the discontinuities of the given functions. If there are none, then write "none"

(1) $f(x) = x^2 - 4$
(2) $f(x) = \frac{x}{x^2-4}$
(3) $f(x) = \frac{x}{x^2+4}$
(4) $f(x) = \tan x$
(5) $f(x) = \frac{\sqrt{4-3x}}{x-2}$
(6) $f(x) = \frac{x-2}{\sqrt{4-3x}}$
(7) $f(x) = \begin{cases} 2 + \frac{3}{x}, & if\ x \leq 1 \\ 2x-1\,, & if\ x > 1 \end{cases}$

Problem 1.7. Find the value of k so that the function f is continuous everywhere

$$f(x) = \begin{cases} 3x+k, & if\ x \leq 2 \\ kx^2\,, & if\ x > 2 \end{cases}$$

Problem 1.8. Find the value of k so that the function f is continuous at x = 0

$$f(x) = \begin{cases} \frac{\tan(6x)}{\tan(3x)}, & if\ x \neq 0 \\ 2k-1\,, & if\ x = 0 \end{cases}$$

Problem 1.9. Given that $\lim\limits_{x\to 1} f(x) = 3$, and $\lim\limits_{x\to 1} h(x) = 0$, find the following limits

a) $\lim\limits_{x\to 1} \frac{h(x)}{f(x)}$
b) $\lim\limits_{x\to 1} \frac{f(x)}{(h(x))^2}$

Problem 1.10. Use the intermediate value theorem to show that the equation $x^3 + x^2 - 2x = 1$ has at least one solution in$[-1, 1]$.

2. Derivatives

Problem 2.1. (i) State the definition of the derivative.
(ii) Use the definition to find the derivative of the following functions
a) $y = \frac{1}{x}$
b) $y = \sqrt{x}$
c) $y = 3x^2 - 5x + 8$

Problem 2.2. Find the derivatives of the following functions:
a) $y = \frac{3x-1}{x^2+7}$
b) $y = e^{x^2} \sin(5x)$
c) $y = \sin^{-1}(x) \ln(3x+1)$
d) $y = \cos^3(7x)$
e) $y = \sin\left(\sqrt{x}\right) + \sqrt{\sin(x)}$
f) $y = x^2 + 2^2 + 2^x$

Problem 2.3. Find the derivatives of the following functions:
a) $y = 4x^3 - 5\cos x - \sec x + \pi^5$
b) $y = \left(x^2 - 3\right) \sin(2x)$.
c) $y = \frac{3x-1}{2x+7}$
d) $y = \sin^3(\tan 5x)$
e) $y = x^5 + 5^x + e^{3x} + \ln(3x) - \ln 7$
f) $y = \sin(3x) + \tan(5x) + \sin^{-1}(3x) + \tan^{-1}(5x)$

Problem 2.4. Consider the curve $y^5 - 2xy + 3x^2 = 9$.
a) Use implicit differentiation to find $\frac{dy}{dx}$
b) Verify that the point (2,1) is on the curve $y^5 - 2xy + 3x^2 = 9$.
c) Give the equation, in slope intercept form of the line tangent to $y^5 - 2xy + 3x^2 = 9$ at the point $(2, 1)$.

Problem 2.5. Use logarithmic differentiation to find the derivatives of the following functions
a) $y = \frac{\sin(x)\sqrt{1+x}}{x^3}$
b) $y = \frac{(x^2+6)\ 3^{2x-1}}{\sqrt[5]{2x-3}}$
c) $y = (3 + \frac{2}{x})^{4x}$

Problem 2.6. Find the derivative of a function y defined implicitly by the given equation
a) $x = \sin(xy)$
b) $x^2 + \sqrt{xy} = 7$
c) $5x^2 - xy - 4y^2 = 0$

Problem 2.7. Find the equation of the tangent line to the given curve at the given point
a) $x^2y + \sin y = 2\pi$ at P(1, 2π)
b) $y = 3x + e^{3x}$ at x = 0
c) $2x^3 - x^2y + y^3 - 1 = 0$ at P(2,-3)

Problem 2.8. Find the second derivative of the following functions
a) $y = \sqrt{2x-3}$
b) $y = (5x-3)^5$
c) $y = \tan(4x)$

Problem 2.9. Given a parametric curve $x = t + \cos t$, $y = 1 + \sin t$. Find $\frac{dy}{dx}$ without eliminating the parameter

3. Local Linear Approximation and differentials

Problem 3.1. Use a local linear approximation of a function in order to find a rational approximation to $\sqrt{37}$.

Problem 3.2. Find the local linear approximation of $f(x) = xsin(x^2)$ at $x_0 = 1$.

Problem 3.3. Find the local linear approximation of $f(x) = \sqrt{x}$ at $x_0 = 4$.

Problem 3.4. (a) Find the local linear approximation of $f(x) = \frac{1}{\sqrt{1-x}}$ at $x_0 = 0$.
(b) Use the linear approximation obtained in part (a) to approximate $\frac{1}{\sqrt{0.99}}$.

Problem 3.5. (a) Find the local linear approximation of $f(x) = e^x \cos(x)$ at $x_0 = 0$.
(b) Use the local linear approximation obtained in part (a) to approximate $e^{0.1} \cos(0.1)$.

Problem 3.6. Use local linear approximation to approximate $\sqrt[3]{64.01}$.

Problem 3.7. Use local linear approximation to approximate $tan(44^{\o})$.

Problem 3.8. The surface area of a sphere is $S = 4\pi r^2$. Estimate the percent error in the surface area if the percent error in the radius is $\pm 3\%$.

Problem 3.9. The surface area of a sphere is $S = 4\pi r^2$. Estimate the percent error in the radius if the percent error in the surface area is $\pm 4\%$.

Problem 3.10. If $y = 2x^2 + 3x + 1$, find Δy and dy.

4. Related Rates

Problem 4.1. A stone dropped into a still pond sends out a circular ripple whose radius increases at a constant rate of 3 ft/s. How rapidly is the area enclosed by the ripple increasing at the end of 10 s?

Problem 4.2. Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of 6 mi^2/h. How fast is the radius of the spill increasing when the area is 9 mi^2?

Problem 4.3. A spherical balloon is inflated so that its volume is increasing at the rate of 3 ft^3/min. How fast is the diameter of the balloon increasing when the radius is 1 ft?

Problem 4.4. A spherical balloon is to be deflated so that its radius decreases at a constant rate of 15 cm/min. At what rate must air be removed when the radius is 9 cm?

Problem 4.5. A 17 ft ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5 ft/s, how fast will the top of the ladder be moving down the wall when it is 8 ft above the ground?

Problem 4.6. A 13 ft ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2 ft/s, how fast will the foot be moving away from the wall when the top is 5 ft above the ground?

Problem 4.7. A 10 ft plank is leaning against a wall. If at a certain instant the bottom of the plank is 2 ft from the wall and is being pushed toward the wall at the rate of 6 in/s, how fast is the acute angle that the plank makes with the ground increasing?

Problem 4.8. A rocket, rising vertically, is tracked by a radar station that is on the ground 5 mi from the launchpad. How fast is the rocket rising when it is 4 mi high and its distance from the radar station is increasing at a rate of 2000 mi/h?

Problem 4.9. A camera mounted at a point 3000 ft from the base of a rocket launching pad. At what rate is the rocket rising when the elevation angle is $\pi/4$ radians and increasing at a rate of 0.2 rad/s?

Problem 4.10. A conical water tank with vertex down has a radius of 10 ft at the top and is 24 ft high. If water flows into the tank at a rate of 20 ft^3/min, how fast is the depth of the water increasing when the water is 16 ft deep?

Problem 4.11. Grain pouring from a chute at the rate of 8 ft^3/min forms a conical pile whose height is always twice its radius. How fast is the height of the pile increasing at the instant when the pile is 6 ft high?

Problem 4.12. _A camera is mounted at a point 3000 feet from the base of a rocket launch pad where a mechanism keeps it aimed at the rocket at all time. At what rate is the camera-to-rocket distance changing when the rocket is 4000 feet up and rising vertically at the rate of 800 feet per second?

Problem 4.13. A boat is pulled into a dock by means of a rope attached to a pulley on the dock.The rope is attached to the boat at a level 10 feet below the pulley.How fast should the rope be pulled if one needs the boat to be approaching the dock at the rate of 12ft/min when there are 125 ft of rope out?

5. Mean-Value Theorem

Problem 5.1. (a) State the Mean-Value Theorem (for a function f on an interval $[a,\ b]$). Be sure to include all assumptions of the theorem and carefully state its conclusion.

(b) Show that the function $f(x) = x^3 + 3x - 4$ satisfies the assumptions of the Mean-Value Theorem on the interval $[0,\ 2]$ and find the value(s) of c in $(0,\ 2)$ satisfying the conclusion of the theorem.

Problem 5.2. (a) State the Mean-Value Theorem (for a function f on an interval $[a,\ b]$). Be sure to include all assumptions of the theorem and carefully state its conclusion.

(b) Show that the function $f(x) = x - \sqrt{x}$ satisfies the assumptions of the Mean-Value Theorem on the interval $[0,\ 4]$ and find the value(s) of c in $(0,\ 4)$ satisfying the conclusion of the theorem.

Problem 5.3. * (a) Let $f(x) = x^{2/3}$ and let $a = -8$, $b = 1$. Show that there is no point c in the interval $(a,\ b)$ so that

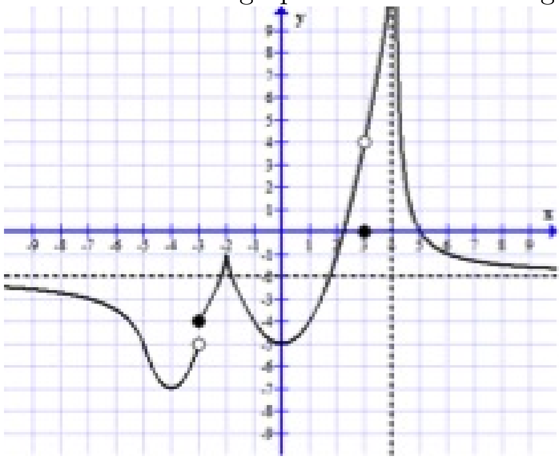
$$f'(c) = \frac{f(b) - f(a)}{b - a} \quad .$$

(b) Explain why the result in part (a) does not contradict the Mean-Value Theorem.

Problem 5.4. Suppose that a state police force has deployed an automated radar tracking system on a highway that has a speed limit of 65 mph. A driver passes through one radar detector at 1:00pm, traveling 60 mph at that moment. The driver passes through a second radar detector 60 miles away, at 1:45pm, again traveling 60 mph at that moment. After a couple of days, the driver receives a speeding ticket in the mail. Argue with the Mean-Value Theorem that the speeding ticket is justified.

6. Monotonicity, Convexity, Local and Global Extrema, Graphs of functions

Problem 6.1. The graph of a function f is given below. Answer the questions that follow.



(i) Find the following limits.

a) $\lim_{x\to-\infty} f(x)$ b) $\lim_{x\to-3^-} f(x)$ c) $\lim_{x\to-3} f(x)$ d) $\lim_{x\to 3} f(x)$ e) $\lim_{x\to 4} f(x)$

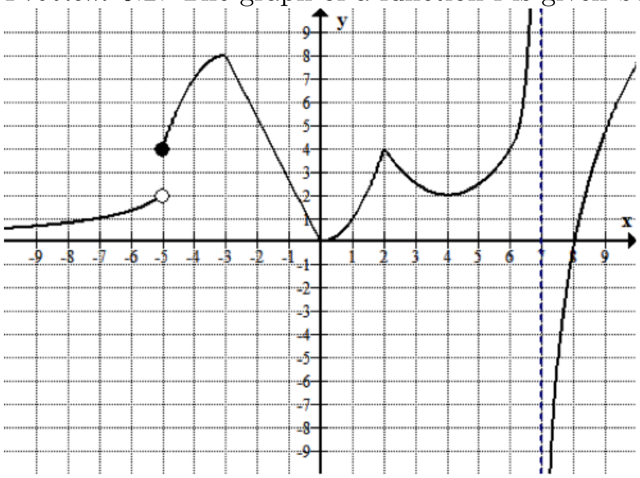
(ii) List the x-value(s) (if any) where f(x) is not continuous.

(iii) List the x-value(s) (if any) where f(x) is continuous but not differentiable.

(iv) Estimate the x coordinate of any inflection point(s).

(v) Does f have absolute extrema? If yes, for what value(s) of x? Identify each extremum as minimum/maximum.

Problem 6.2. The graph of a function f is given below. Answer the questions that follow.



(i) Find the following limits

(1) $\lim_{x\to-\infty} f(x)$

(2) $\lim_{x\to-5^-} f(x)$

(3) $\lim_{x\to-5^+} f(x)$

(4) $\lim_{x\to-5} f(x)$

(5) $\lim_{x\to2} f(x)$

(6) $\lim_{x\to7^-} f(x)$

(7) $\lim_{x\to7} f(x)$

(8) $\lim_{x\to+\infty} f(x)$

(ii) Is this function continuous everywhere? If not, at what points (give values of x). At each point state which condition of continuity fails? (Function has a jump discontinuity is **not** a reason)
(iii) Does f have any asymptote(s)? If yes, what kind? Write their equations.
(iv) On what intervals is f decreasing?
(v) On what intervals is f concave down?
(vi) What is $f'(-2)$?
(vii) At what value(s) x, if any, does f have a relative maximum?
(viii) List values of x at which f is not differentiable.

Problem 6.3. For each function $y = f(x)$:

- Find the domain and the limits at the endpoints of the domain.
- Find $f'(x)$
- Deduce the intervals where f is increasing and the intervals where f is decreasing,
- List local extrema of f
- Find $f''(x)$
- Deduce the intervals where f is concave up and the intervals where it is concave down
- List inflection points (if any)
- Sketch the graph of f.

(1) $f(x) = 2x^3 + x^2 - 20x + 1$

(2) $f(x) = 3x^4 - 4x^3$

Problem 6.4. For each function find the intervals where the function is increasing, the intervals where it is decreasing, the vertical and horizontal asymptotes if any, and sketch the graph.

(1) $f(x) = \dfrac{1}{x^2 - 9}$

(2) $f(x) = \dfrac{8x}{x^2 + 4}$

Problem 6.5. In each part sketch a continuous curve $y = f(x)$ with the stated properties.

(1) $f(3) = 5, \quad f'(3) = 0, \quad f''(x) < 0 \text{ for } x < 3 \quad \text{and} \quad f''(x) > 0 \text{ for } x > 3$

(2) $f(2) = 4, \quad f''(x) < 0 \text{ for } x \neq 2, \quad \text{and} \lim_{x \to 2^+} f(x) = -\infty \quad \lim_{x \to 2^-} f(x) = +\infty$

Problem 6.6. Let $f(x) = x \ln x$. Find $\lim_{x \to 0^+} f(x)$, $\lim_{x \to \infty} f(x)$. Sketch the graph $f(x)$. Note that $f'(x) = 1 + \ln x$.

Problem 6.7. Let $f(x) = x\,\mathrm{e}^{-x}$. Find $\lim_{x \to -\infty} f(x)$, $\lim_{x \to \infty} f(x)$. Use the fact that $f'(x) = (1 - x)\,\mathrm{e}^{-x}$ and $f''(x) = (x - 2)\mathrm{e}^{-x}$ to sketch the graph of f

Problem 6.8. Let $f(x) = \mathrm{e}^{-x^2/2}$. Find $\lim_{x \to -\infty} f(x)$, $\lim_{x \to \infty} f(x)$. Use the fact that $f'(x) = -x\,e^{-x^2/2}$ and $f''(x) = (x^2 - 1)\mathrm{e}^{-x^2/2}$ to sketch the graph of f

7. Optimization

Problem 7.1. There are 280 yards of fencing available to enclose a rectangular field. How should this fencing be used so that the enclosed area is as large as possible?

Problem 7.2. An apartment complex has 520 apartments. At $1000 per month for each apartment all the apartment will be occupied. Each $100 increase will produce 20 vacancies. What should the rent be to generate the largest revenue?

Problem 7.3. We need to enclose a field with a rectangular fence. We have 100 ft of fencing material and a building is on one side of the field and so won't need any fencing. Determine the dimensions of the field that will enclose the largest area.

Problem 7.4. We want to construct a box with a square base and we only have 10 m^2 of material to use in construction of the box. Assuming that all the material is used in the construction process determine the maximum volume that the box can have.

Problem 7.5. A printer needs to make a poster that will have a total area of 150 in^2 and will have 1 in margins on the sides, a 2 in margin on the top and a 1 in margin on the bottom. What dimensions of the poster will give the largest printed area?

Problem 7.6. The regular air fare between Miami and Chicago is $450. An airline using planes with a capacity of 300 passengers on this route observes that they fly with an average of 175 passengers. Market research tells the airlines' managers that each $10 fare reduction would attract, on average, 5 more passengers for each flight. How should they set the fare to maximize their revenue?

Problem 7.7. A Florida Citrus grower estimates that if 50 orange trees are planted; the average yield per tree will be 300 oranges. The average yield will decrease by 5 oranges per tree for each additional tree planted on the same acreage. How many trees should the grower plant to maximize the total yield?

Problem 7.8. A rectangle has its two lower corners on the x-axis and its two upper corners on the curve $y = 27 - x^2$. For all such rectangles, what are the dimensions of the one with largest area?

Problem 7.9. An open-top rectangular box is constructed from a 10-in by 16-in piece of cardboard by cutting squares of equal side length from the corners and folding up the sides. Find the dimensions of the box of largest volume.

Problem 7.10. A rectangular plot of land is to be fenced in using two types of fencing. Two opposite sides will use heavy-duty fencing selling for $3 a foot. The two remaining sides will use standard fencing selling for $2 a foot. What are the dimensions of the rectangular plot of greatest area that can be fenced in at a cost of $6000?

Problem 7.11. For each function on the indicated interval, find the absolute maximum and absolute minimum, if these exist. If one or both do not exist, specify so.

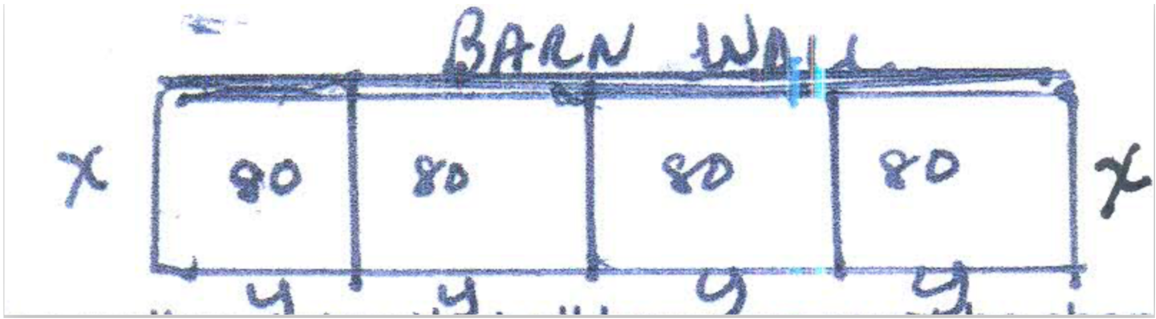
(a) $f(x) = 2x^5 - 5x^4 + 7$ on $[-1, 3]$

(b) $f(x) = x^{1/3}(x+4)$ on $[-1, 3]$

(c) $f(x) = x + \dfrac{1}{x}$ on $(0, +\infty)$

(d) $f(x) = x^2 e^{2x}$ on $(-\infty, 0]$

Problem 7.12. A rancher plans to make four identical and adjacent rectangular pens against a barn, each with 80 square meter area. What are the dimensions of each pen that minimize the amount of fence that must be used? (Along the barn, no fence is needed.)



Problem 7.13. Suppose an airline policy states that all baggage must be shaped with the sum of the length, width and height not to exceed 108 inches. What are the dimensions of a **square** based box with greatest volume under this condition?

Problem 7.14. A closed rectangular container with a square base is to have a volume of 2250 cubic inches. The material for the top and the bottom of the container will cost \$2 per square inch, and the material for the sides will cost \$3 per square inch. Find the dimensions of the container of least cost.

Problem 7.15. You are asked to make a cylindrical can with a given volume of 81π cm^3. Find the dimensions of the can, radius and height, that will minimize the amount of material used. Assume the same material is used for the top and bottom of the can as for the curved side.

Hint: You are given the volume, you have to minimize the total surface area.

Problem 7.16. Car B is 30 miles directly east of Car A and begins moving west at 90 mph. At the same moment car A begins moving north at 60 mph. What will be the minimum distance between the cars and at what time t does the minimum distance occur ?

Hint: Choose a good coordinate system and minimize the square of the distance.

8. True/False questions

For each of the following, answer whether the statement is True of False and then give a brief justification of your answer. Such questions may appear on your exam, with or without the justification requirement.

Problem 8.1. If $\lim_{x\to 2} f(x) = 3$ and $\lim_{x\to 2} g(x) = 2$ then $\lim_{x\to 2}\left(\frac{1}{f(x)} + \frac{1}{g(x)}\right) = \frac{5}{6}$. **True** **False**

Justification:

Problem 8.2. If $\lim_{x\to 2} f(x) = 0$ and $\lim_{x\to 2} g(x) = 0$ then $\lim_{x\to 2} \frac{f(x)}{g(x)} = 0$. **True** **False**

Justification:

Problem 8.3. If $\lim_{x\to 2} f(x) = 0$ and $\lim_{x\to 2} g(x) = 0$ then $\lim_{x\to 2} \frac{f(x)}{g(x)}$ does not exist. **True** **False**

Justification:

Problem 8.4. $y = \tan x$ is continuous on the whole real line. **True** **False**

Justification:

Problem 8.5. Every polynomial is continuous on the whole real line. **True** **False**

Justification:

Problem 8.6. Every rational function is continuous at all points at which it is defined. **True** **False**

Justification:

Problem 8.7. A function can never cross its horizontal asymptote. **True** **False**

Justification:

Problem 8.8. If $f(x)$ is continuous everywhere then $|f(x)|$ is continuous everywhere. **True** **False**

Justification:

Problem 8.9. If $p(x)$ is a polynomial function and $p(1) = 7$, $p(3) = -5$, then there is a value c in the interval $(1,\ 3)$ so that $p(c) = 0$. **True** **False**

Justification:

Problem 8.10. * If $p(x)$ is a polynomial function and $p(0) = p(1) = 2017$, then there is no value c in the interval $(0,\ 1)$ so that $p(c) = 0$. **True** **False**

Justification:

Problem 8.11. If a function satisfies $|f(x) - 5| \leq 7|x - 3|$ for all real numbers x, then $\lim\limits_{x\to 3} f(x) = 5$. **True** **False**

Justification:

Problem 8.12. $\lim\limits_{x\to 0^+} \ln x = -\infty$ **True** **False**

Justification:

Problem 8.13. $\frac{\sin x}{x} = 1$ **True** **False**

Justification:

Problem 8.14. $\lim\limits_{x\to +\infty} \dfrac{\sin x}{x} = 0$ **True** **False**

Justification:

Problem 8.15. $f'(x) = \frac{f(x+h) - f(x)}{h}$ **True** **False**

Justification:

Problem 8.16. If a function f is continuous at $x = 0$, then f is differentiable at $x = 0$. **True** **False**

Justification:

Problem 8.17. If $y = e^{kx}$, where k is a constant, then $dy/dx = ky$. **True** **False**

Justification:

Problem 8.18. If $g(x) = f(x)\sin x$ then $g'(x) = f'(x)\cos x$. **True** **False**

Justification:

Problem 8.19. If $g(x) = f(\sin x)$ then $g'(x) = f'(\cos x)$. **True** **False**

Justification:

Problem 8.20. $\tan^{-1} x = \cot x$ **True** **False**

Justification:

Problem 8.21. If $p(x)$ is a degree 3 polynomial, then its fourth derivative is zero. **True** **False**

Justification:

Problem 8.22. If $f'(2) = 0$ then f has a relative minimum or a relative maximum at $x = 2$. **True** **False**

Justification:

Problem 8.23. If $f'(2) = 0$ and $f''(2) < 0$ then f has a relative maximum at $x = 2$. **True** **False**

Justification:

Problem 8.24. If f is continuous on (a, b) and $\lim_{x\to a^+} f(x) = -\infty$, then f has no absolute minimum on (a, b). **True** **False**

Justification:

Problem 8.25. If $f(x)$ is differentiable on the interval $[a, b]$ and $f'(x) < 0$ for all x in $[a, b]$, then on the interval $[a, b]$, f has an absolute maximum at $x = a$. **True** **False**

Justification:

Problem 8.26. $y = x^{1/3}$ has a vertical tangent line at $x = 0$. **True** **False**

Justification:

Problem 8.27. If $f(x)$ is differentiable for all x and $f(2) = f(3)$ then there is a point $2 < c < 3$ where $f'(c) = 0$. **True** **False**

Justification:

Problem 8.28. An antiderivative of $\ln(x)$ is $1/x$. **True** **False**

Justification:

Problem 8.29. If f is increasing on $(-2, 3)$ then $f(0) \leq f(1)$. **True** **False**

Justification:

Problem 8.30. If $\lim\limits_{x \to 1^-} f(x) = +\infty$ then $x = 1$ is a vertical asymptote for f. **True** **False**

Justification:

9. Antiderivatives

Evaluate the following antiderivatives.

Problem 9.1. $\displaystyle\int (x^2 - 3)\,dx$

Problem 9.2. $\displaystyle\int (x^{-3} - 3\sqrt[4]{x} + 8\sin x)\,dx$

Problem 9.3. $\displaystyle\int \Big(2^x + \frac{1}{2x}\Big)\,dx$

Problem 9.4. $\displaystyle\int \Big(\frac{1}{\sqrt{1-x^2}} - 4\sec x\tan x\Big)\,dx$

Problem 9.5. $\displaystyle\int (e^x + x^e)\,dx$

Problem 9.6. $\displaystyle\int (1+2x^2)^2\,dx$

Problem 9.7. $\displaystyle\int \frac{x^5+2x^3-1}{x^4}\,dx$

Problem 9.8. $\displaystyle\int \frac{x+1}{\sqrt{x}}\,dx$

Problem 9.9. $\displaystyle\int \sec x(\sec x + \tan x)\,dx$

Problem 9.10. $\displaystyle\int 2x(x^2+1)^{23}\,dx$

Problem 9.11. $\displaystyle\int x^3\sqrt{x^4+1}\,dx$

Problem 9.12. $\displaystyle\int \cos^3 x\sin x\,dx$

Problem 9.13. $\displaystyle\int e^{\tan(2x)}\sec^2(2x)\,dx$

Problem 9.14. $\displaystyle\int (4x-3)^9\,dx$

Problem 9.15. $\displaystyle\int \frac{1}{1-3x}\,dx$

Problem 9.16. $\displaystyle\int \frac{1}{x\ln x}\,dx$

Problem 9.17. $\displaystyle\int \frac{1}{1+4x^2}\,dx$

Problem 9.18. $\displaystyle\int \frac{x}{1+4x^2}\,dx$

Problem 9.19. $\displaystyle\int \frac{e^x}{\sqrt{1-e^{2x}}}dx$

Problem 9.20. $\displaystyle\int \frac{\cos(5/x)}{x^2} dx$

Problem 9.21. $\displaystyle\int \frac{x^2}{e^{x^3}}\, dx$

Problem 9.22. $\displaystyle\int \frac{x}{\sqrt{4-x}} dx$

Problem 9.23. Solve the initial value problem $\dfrac{dy}{dx} = 6e^x, \quad y(0) = 2$

Problem 9.24. Solve the initial value problem $\dfrac{dy}{dx} = \sqrt{x}(6+5x), \quad y(1) = 5$

Problem 9.25. Find the most general form of $f(x)$ if $f''(x) = \sqrt[3]{x} + 1$. (Hint: Integrate twice.)